北村記念シンポジウム

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Equilibrium Properties of Taxi Market with Search Frictions

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Taxi research by Kitamura Sensei and his colleagues:

Kitamura, R. (2005) Taxis in the 21st century: Public transport in a motorized society. *Transport Economics Studies Review*, 8 (2), 111 (in Japanese).

Kitamura, R. and Yoshii, T. (2005) Rationality and heterogeneity in taxi driver decision: An application of a stochastic-process model of taxi behavior. In H.S. Mahmassani (ed.) *Transportation and Traffic Theory: Flow, Dynamics and Human Interaction*, Elsevier, Oxford, 609-628.

Previous Studies on the Economics of Taxi Services By Economists

$$D = D(F, W), \quad \frac{\partial D}{\partial F} < 0, \quad \frac{\partial D}{\partial W} < 0$$

$$W = W(V), \quad \frac{\partial W}{\partial V} < 0$$

$$TC = c(Q + V)$$

D = Demand for taxi ride

Q = Occupied taxi-hour

V = Vacant taxi-hours

F = Expected price of a taxi ride

W = Expected customer waiting time

c = cost per taxi hour of service time

TC = Total taxi operating cost

Key assumption: ***Customer waiting time is negatively related to the number of vacant taxis (Douglas, 1972; De vany, 1975; Hackner and Nyberg, 1995; Arnott, 1996; Cairns and Liston-Heyes, 1996)***

Previous Studies on the Economics of Taxi Services By Economists

- Social Optimum (First-best) Solution
- Second-best Solution (welfare maximization under zero-profit constraint)
- Monopoly Solution (profit maximization)
- Competition and Regulation (price control and entry restriction)

Two Major Findings in the literature

- *Taxi services should be subsidized in a first-best environment*

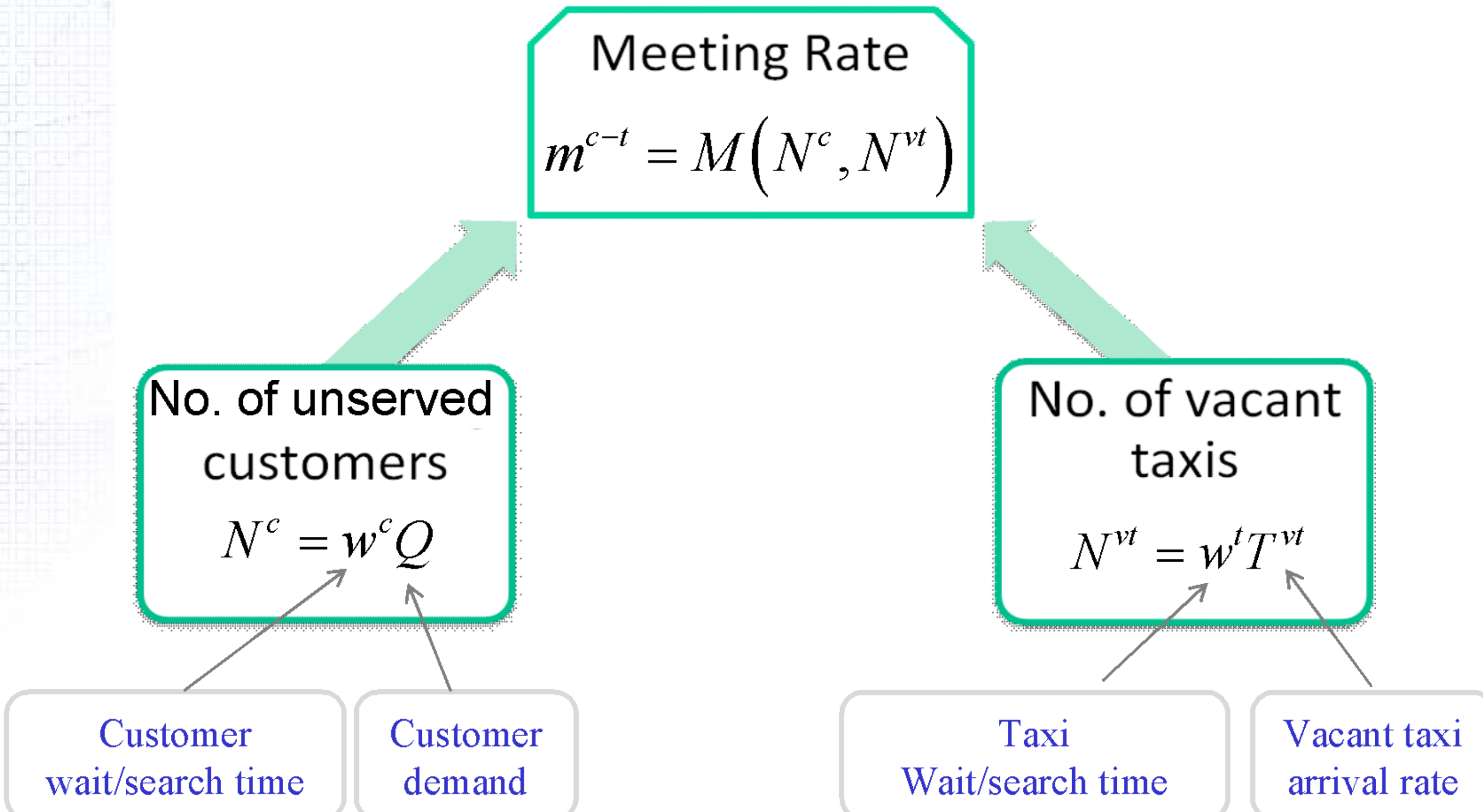
  The total taxi profit is negative at social optimum due to the increasing returns to scale in taxi service production (Douglas, 1972; Arnott, 1996).

- *Existence of Pareto-improvement in both taxi service quality and market profitability*

  Within a certain region, an increase in the number of taxis can result in both a reduction in expected customer waiting times and an increase in taxi utilization rates and the resulting average net revenue per taxi (Manski and Wright, 1976; Schroeter, 1983; Yang et al., 2005).

Bilateral Taxi-Customer Searching and Meeting Function

In cruising taxi markets:



Returns to Scale in the Meeting Function

$$m^{c-t} = M\left(N^c, N^{vt}\right) \qquad \alpha_1 = \frac{\partial M}{\partial N^c}\frac{N^c}{M} \qquad \alpha_2 = \frac{\partial M}{\partial N^{vt}}\frac{N^{vt}}{M}$$

$$0 < \alpha_1, \alpha_2 \le 1$$

α_1: elasticity of meeting rate with respect to the number of wait/search customers

α_2: elasticity of meeting rate with respect to the number of vacant taxis

Cobb-Douglas type meeting function:

$$M\left(N^c, N^{vt}\right) = A\left(N^c\right)^{\alpha_1}\left(N^{vt}\right)^{\alpha_2}$$

A: a constant parameter

Returns to Scale in the Meeting Function

$$m^{c-t} = M\left(N^{c}, N^{vt}\right) = A\left(N^{c}\right)^{\alpha_1}\left(N^{vt}\right)^{\alpha_2}$$

Increasing Returns to Scale (IRS)	$\alpha_1 + \alpha_2 > 1.0$
Constant Returns to Scale (CRS)	$\alpha_1 + \alpha_2 = 1.0$
Decreasing Returns to Scale (DRS)	$\alpha_1 + \alpha_2 < 1.0$

$\alpha_1 = \alpha_2$ Symmetric meeting function

$\alpha_1 \neq \alpha_2$ Asymmetric meeting function with an asymmetric factor α_2/α_1

Basic Definitions

- **Customer Demand:** $Q = f\left(\underbrace{P + \beta w^c + \tau l}_{\text{full trip price}}\right)$

P:	taxi fare per ride
β :	customers' value of waiting time
w^c :	expected customer waiting time
τ :	customers' value of in-taxi ride time
l:	average duration of taxi ride

- **Customer Waiting Time, *w^c*:** the expected customer waiting time is an important service quality measure.

- **Taxi Utilization Rate, *U*:** measured as the fraction of occupied taxi service time. the expected taxi utilization rate governs directly the market profitability.

Pareto-Improving Service Regime

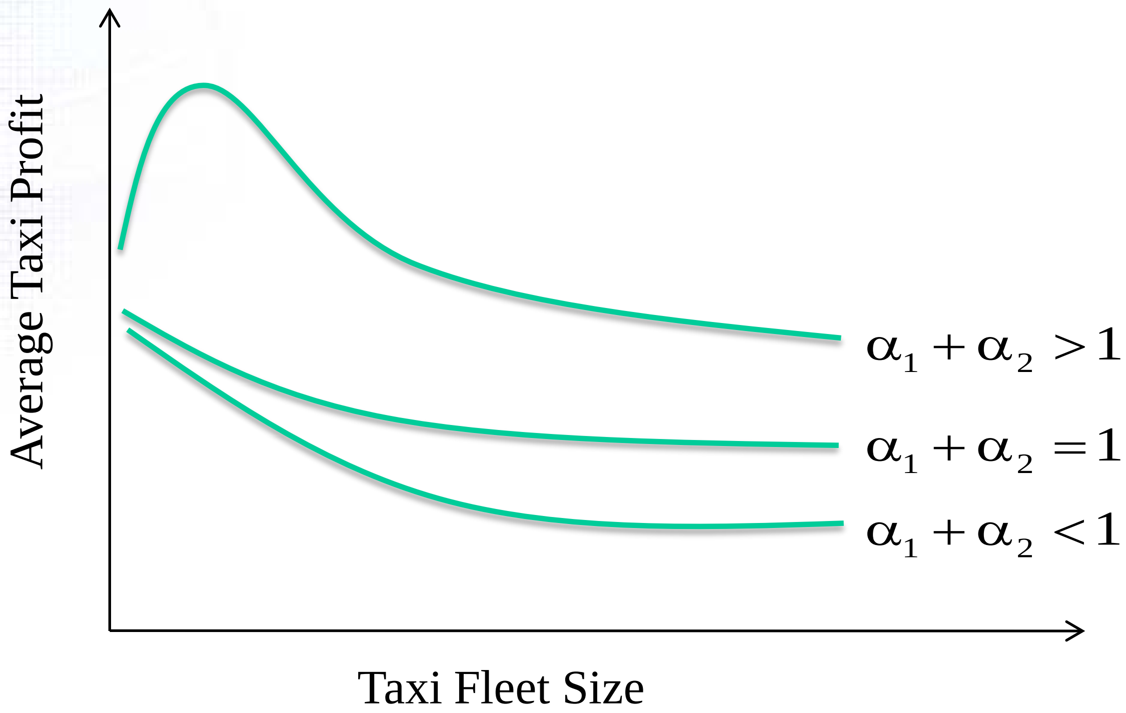
Market Profitability and Service Quality:

Taxi drivers ↔ Average taxi profit

Customers ↔ Customer waiting time or service quality

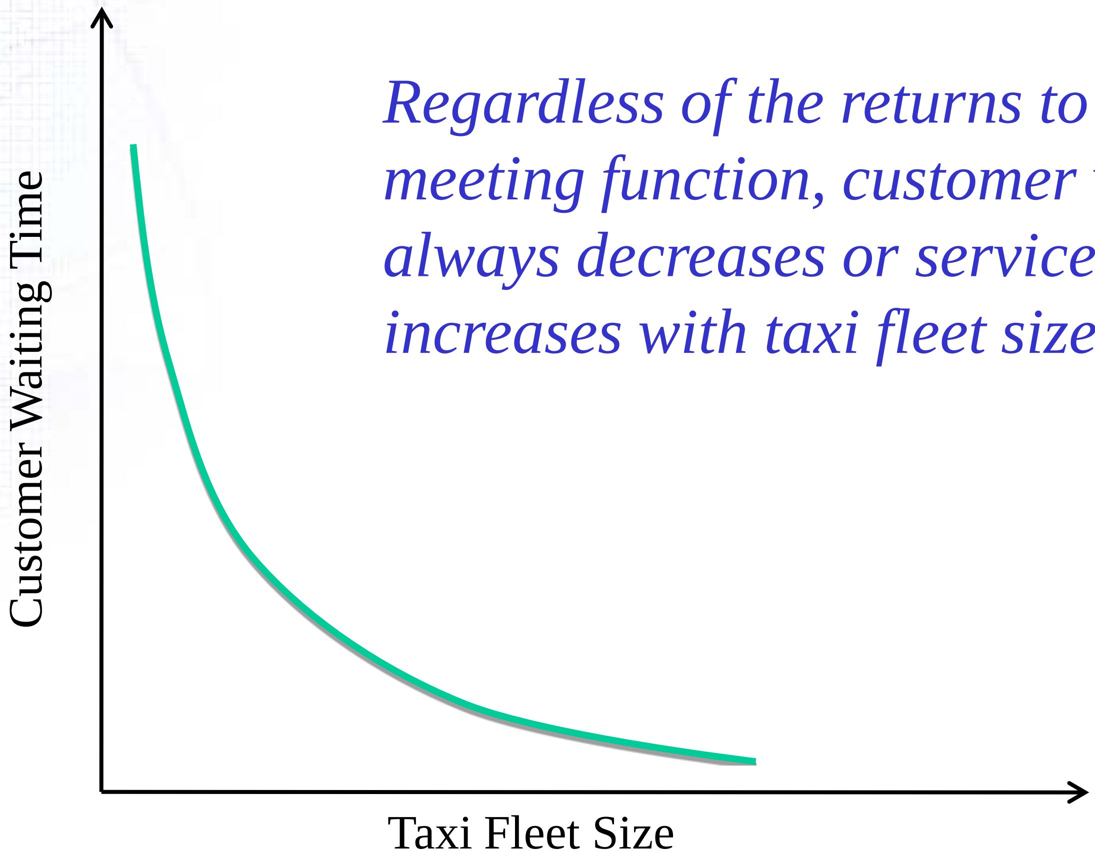
Pareto-Improving Regime

Proposition 1. *The average taxi profit is always decreasing with taxi fleet size when there are constant or decreasing returns to scale in the meeting function; average taxi profit first increases and then decreases with taxi fleet size if and only if there are increasing returns to scale in the meeting function.*



Pareto-Improving Regime

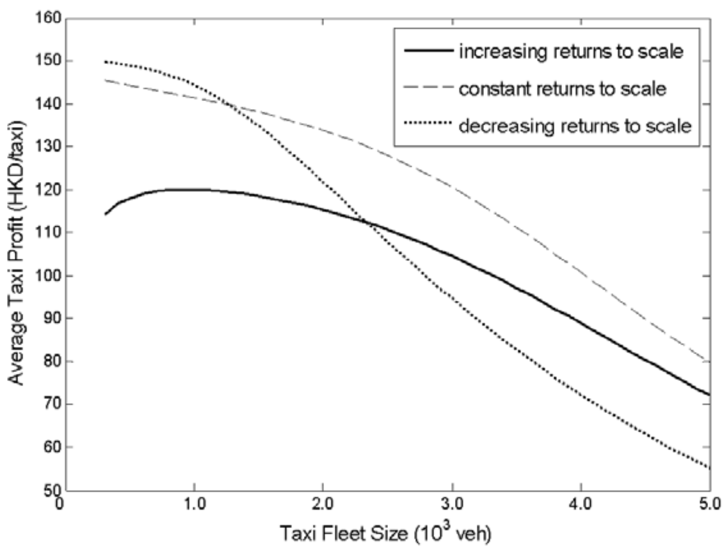
Customer Waiting Time or Service Quality (without traffic congestion):

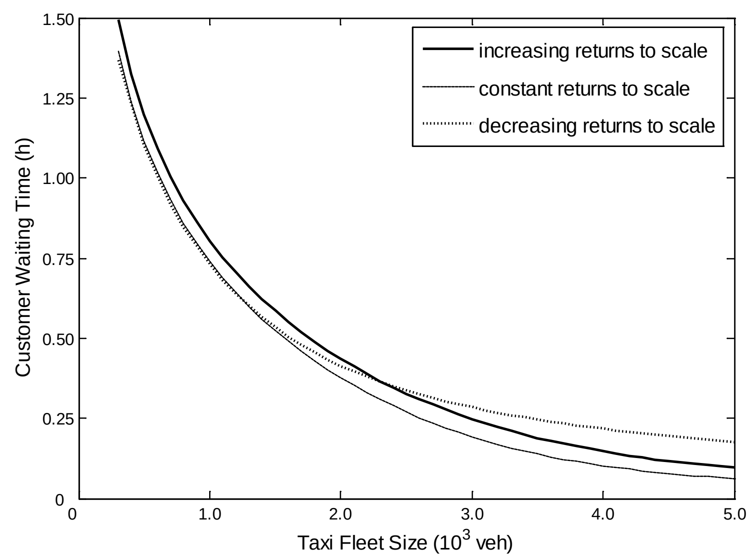


Regardless of the returns to scale in the meeting function, customer waiting time always decreases or service quality always increases with taxi fleet size.

Pareto-Improving Regime

Proposition 2. *A Pareto-improving win-win situation in a taxi market in terms of increase in average taxi profit and decrease in average customer waiting time with taxi fleet size can occur if and only if the bilateral customer-taxi meeting function exhibits increasing returns to scale.*





Properties of Social Optimum Solution

Social optimum problem: to choose optimal taxi fleet size and taxi fare to maximize social welfare.

Social Welfare = Consumer Surplus + Producer Surplus

Properties of Social Optimum Solution

Proposition 3. *At social optimum, taxi fleet size should be determined such that the total taxi operating cost equals the sum of the total occupied taxi operating cost and the total customer waiting time cost multiplied by a factor* α_2/α_1 *, or equivalently, total vacant taxi operating cost equals total customer waiting time cost multiplied by a factor* α_2/α_1 *; taxi fare should be equal to the full cost for serving each taxi ride plus a margin that is positive, zero and negative for the meeting function of decreasing, constant and increasing returns to scale, respectively.*

Properties of Social Optimum Solution

Proposition 4. *At social optimum, taxi profits are negative, zero and positive when the meeting function exhibits increasing, constant and decreasing returns to scale, respectively.*

REMARKS:

Taxi operation at social optimum incurs a loss when there are increasing returns to scale in the meeting function, and hence taxi service should be subsidized.

$\alpha_1 = 1.0$

$0 < \alpha_2 \leq 1.0$

→ Assumption that customer waiting time is made a function solely of the vacant taxi-hours in a negative manner

In this case the cost recovery ratio in equals the taxi utilization rate and thus gives the same result as obtained in previous studies. Namely, at social optimum, taxi revenue covers only the cost of occupied taxi time and as a result taxi services should be subsidized and the subsidy should cover the cost of vacant taxi time.

Taxi Cost Recovery Ratio at Social Optimum

$$\eta = \frac{\text{Total Taxi Revenue}}{\text{Total Taxi Service Cost}} = \frac{PQ}{cN}$$

$$\eta_{so}^{*} = 1 + \frac{1 - \alpha_1 - \alpha_2}{\alpha_2}\left(1 - U_{so}^{*}\right)$$

IRS ($\alpha_1 + \alpha_2 > 1$)	$0 < \eta < 1$
CRS ($\alpha_1 + \alpha_2 = 1$)	$\eta = 1$
DRS ($\alpha_1 + \alpha_2 < 1$)	$\eta > 1$

In the special case:

$\alpha_1 = 1.0$; $0 \le \alpha_2 \le 1$

(adopted in most previous studies by assuming customer waiting time being inversely proportional to total number of vacant taxis):

$$\eta_{so}^{*} = U_{so}^{*}$$

At social optimum, the cost recovery ratio is exactly equal to the taxi utilization rate.

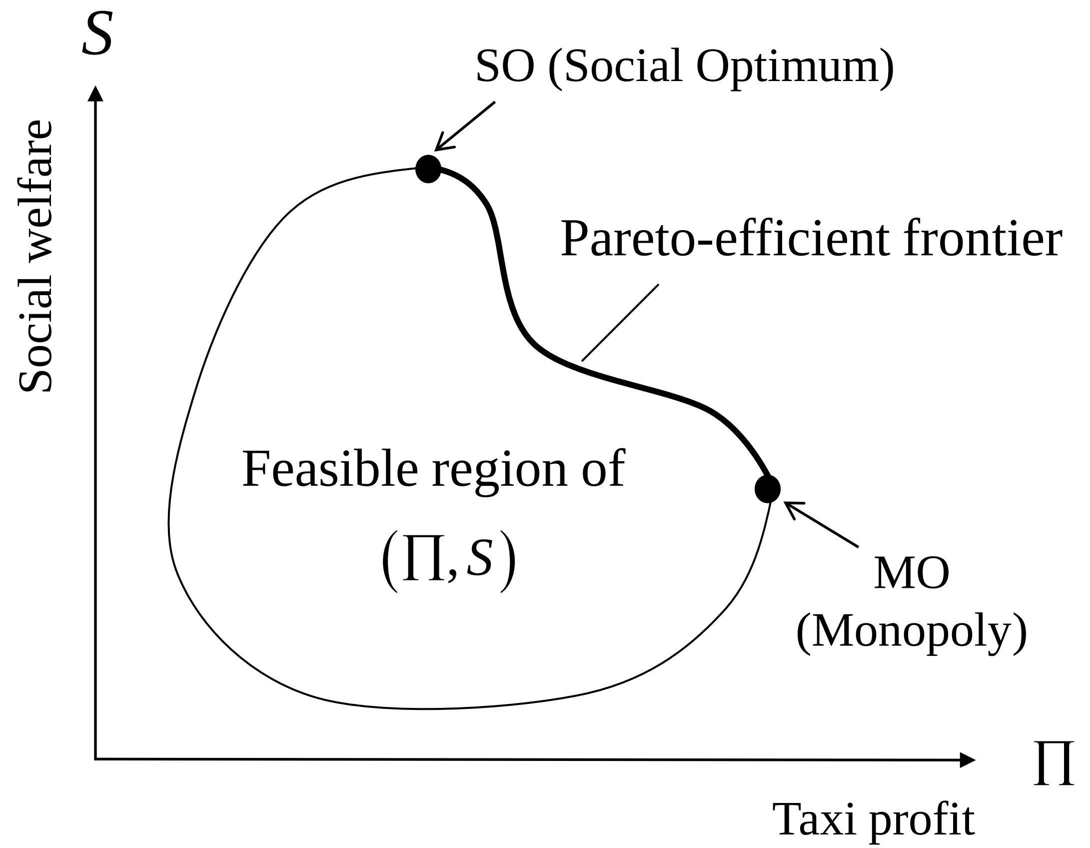
Pareto-Efficient Solution of Maximizing Social Welfare versus Maximizing Total Taxi Profit

$$\max_{(P,N)\in\Omega} \begin{pmatrix} S(P,N) \\ \Pi(P,N) \end{pmatrix}$$

$$\Omega = \{(P,N) : P \geq 0, N \geq 0\}$$

P: taxi fare

N: taxi fleet size



Properties of Pareto Efficient Solution

Proposition 5. *With constant returns to scale in the meeting function,* $\alpha_1 + \alpha_2 = 1.0$, *the taxi utilization rate and taxi service quality (customer waiting time) are constant along the Pareto-efficient frontier and equal to those at social optimum*

$$U^* = \left(1 + \left(\frac{\alpha_2 \beta \zeta}{\alpha_1 c l}\right)^{\alpha_1}\right)^{-1} = const.$$

$$W^{c*} = \frac{\alpha_1 c l}{\alpha_2 \beta}\left(\frac{1}{U^*} - 1\right) = const.$$

Economic Reason: The monopolist tends to offer a smaller taxi fleet size and higher fare charge and hence produces a lower output of customer demand, yielding a taxi utilization ratio that happens to be identical with that at social optimum, where a social planner, by contrast, tends to offer a larger fleet size and lower fare charge and hence produces a higher output of customer demand.

Self-internalization of bilateral search externality: An additional vacant taxi imposes a negative externality on other vacant taxis and a positive externality on unserved customer. An additional unserved customer imposes a negative externality on other unserved customers and a positive externality on vacant taxis. Proposition 5 states that the search externalities offset one another along the Pareto-efficient frontier when the meeting function exhibits constant returns to scale

Properties of Pareto Efficient Solution

Proposition 7. *With non-constant returns to scale in the meeting function,* $\alpha_1 + \alpha_2 \neq 1.0$, *both taxi utilization rate and customer waiting time vary along the Pareto-efficient frontier from SO to MO, depending on the returns to scale in the meeting function.*

i) With increasing returns to scale, taxi utilization rate decreases and customer waiting time increases along the Pareto-efficient frontier from SO to MO.

$$\alpha_1 + \alpha_2 > 1.0 \quad \Rightarrow \quad U_{so}^{*} > U_{mo}^{*}, \; w_{so}^{c*} < w_{mo}^{c*}$$

ii) With decreasing returns to scale, taxi utilization ratio increases and customer waiting time decreases along the Pareto-efficient frontier from SO to MO.

$$\alpha_1 + \alpha_2 < 1.0 \quad \Rightarrow \quad U_{so}^{*} < U_{mo}^{*}, \; w_{so}^{c*} > w_{mo}^{c*}$$

Numerical Example

Demand Function:

$$Q = f\left(P + \beta w^{c} + \tau l\right) = \tilde{Q}\exp\left\{-\kappa\left(P + \beta w^{c} + \tau l\right)\right\},\ \ (\kappa > 0)$$

Cobb-Douglas type meeting function:

$$m^{c-t} = A\left(N^{c}\right)^{\alpha_1}\left(N^{vt}\right)^{\alpha_2}$$

Numerical Experiment

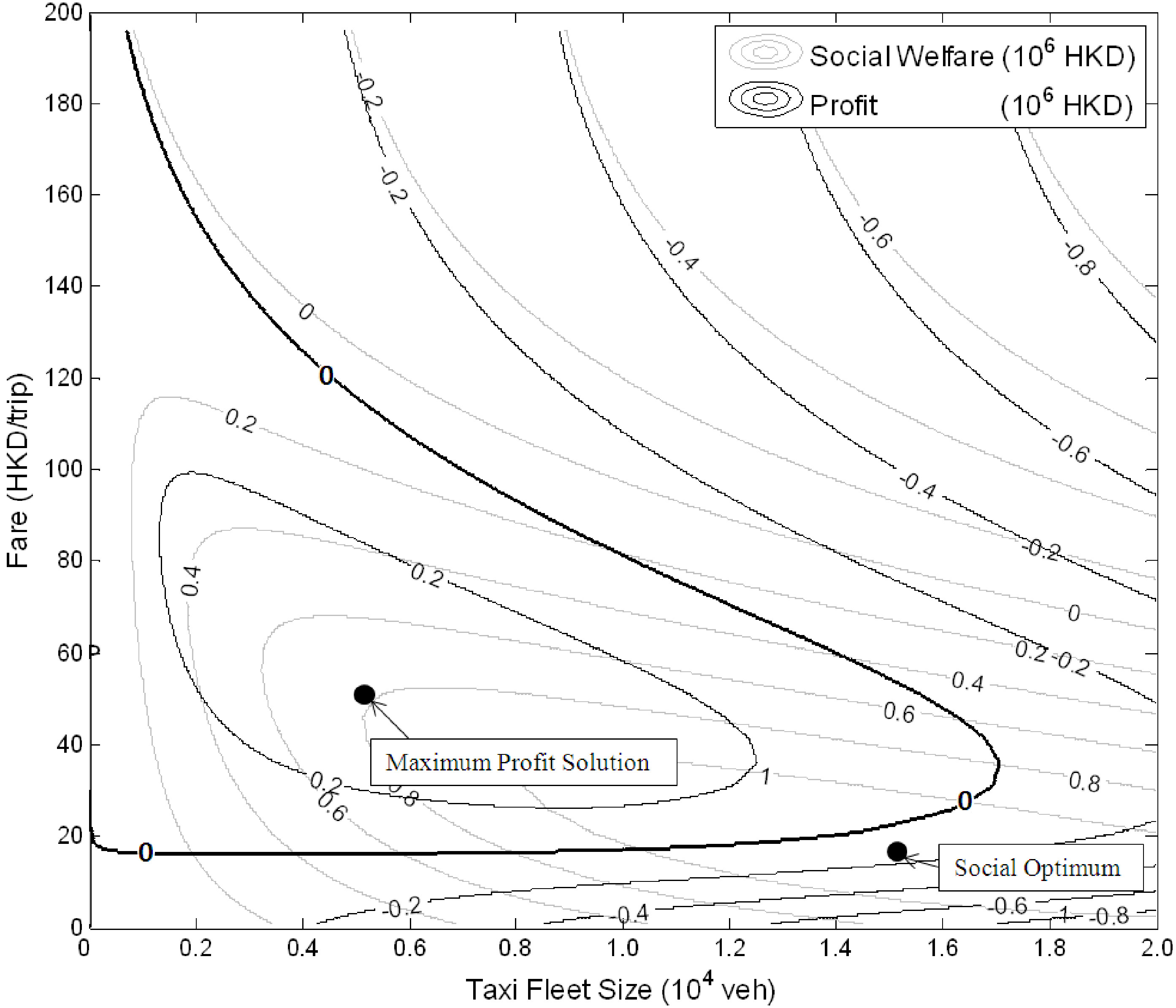
Constants	Notation	Unit	Value
Potential customer demand	$\tilde{Q}$	(trip/h)	1.0×10^5
Demand sensitivity parameter	κ	(1/HKD)	0.03
Customer value of waiting time	β	(HKD/h)	60
Customer value of in-taxi ride time	τ	(HKD/h)	35
Average taxi ride time	l	(h)	0.3
Taxi operating cost per unit time	c	(HKD/h)	50

increasing return to scale: $\alpha_1 = \alpha_2 = 0.75$ $A = 0.2$

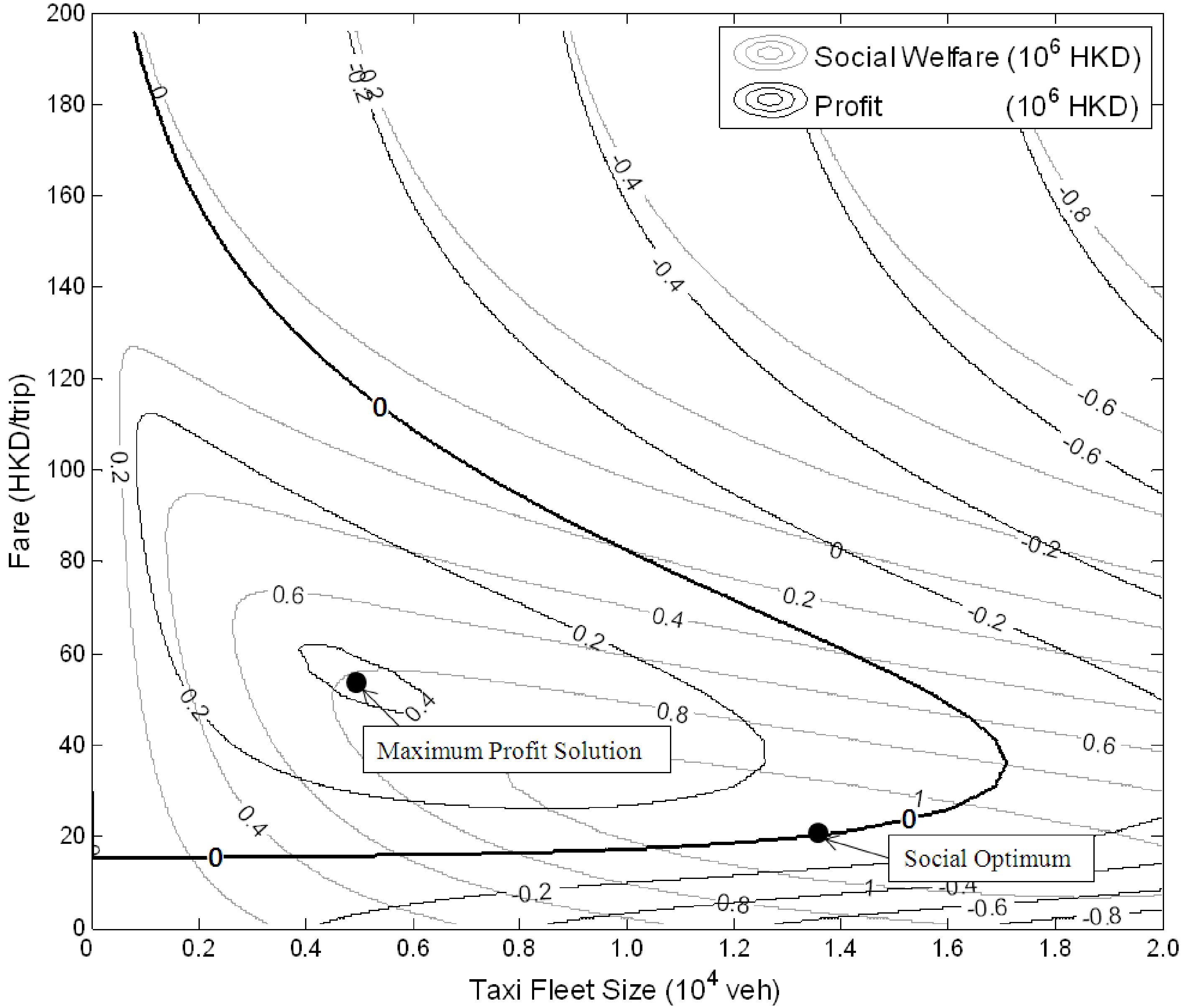
constant return to scale: $\alpha_1 = \alpha_2 = 0.50$ $A = 10.0$

decreasing returns to scale: $\alpha_1 = \alpha_2 = 0.25$ $A = 200.0$

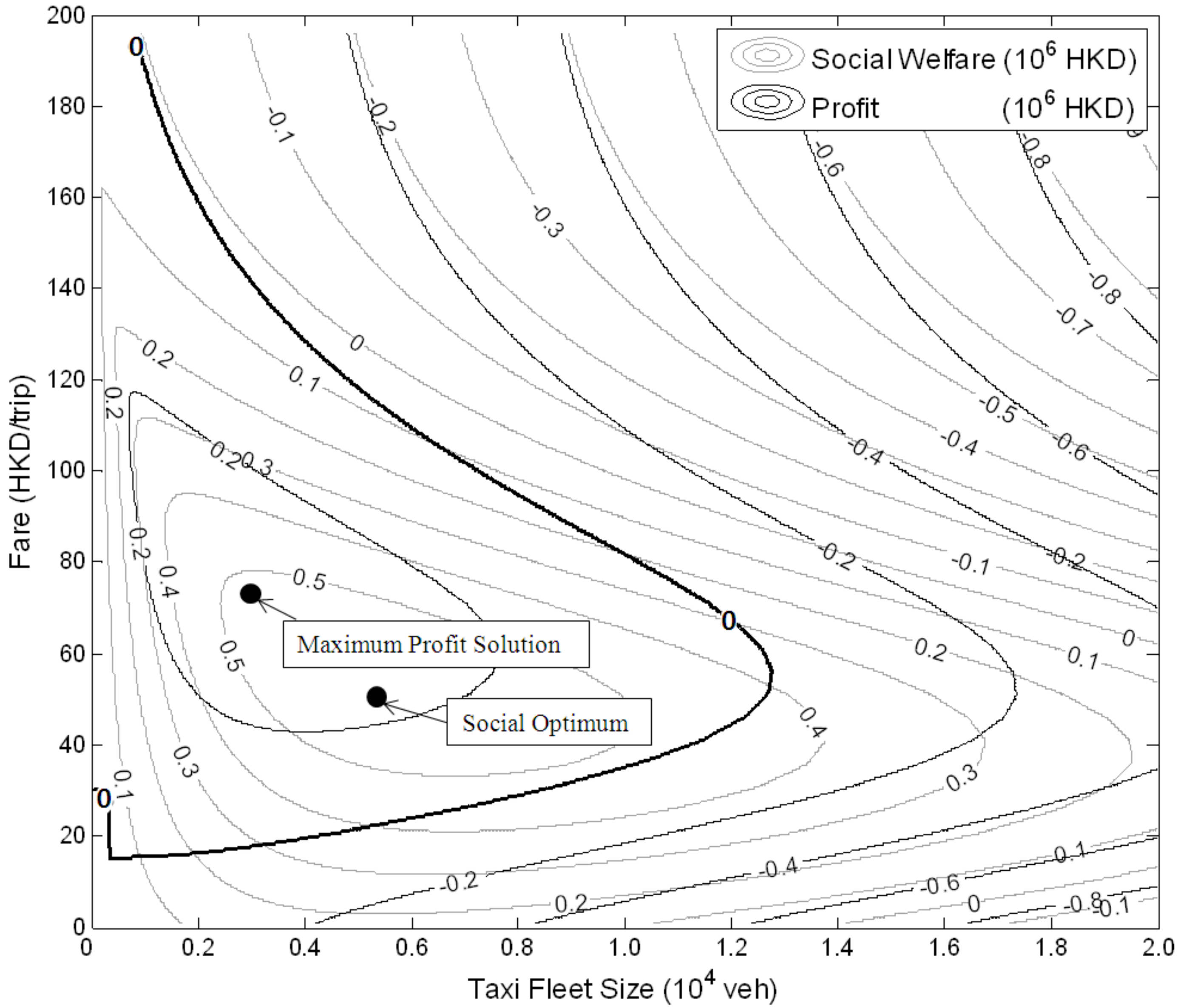
Iso-Social Welfare and Iso-Profit Contours with Increasing Returns to Scale (A=0.2; α_1=α_2=0.75)



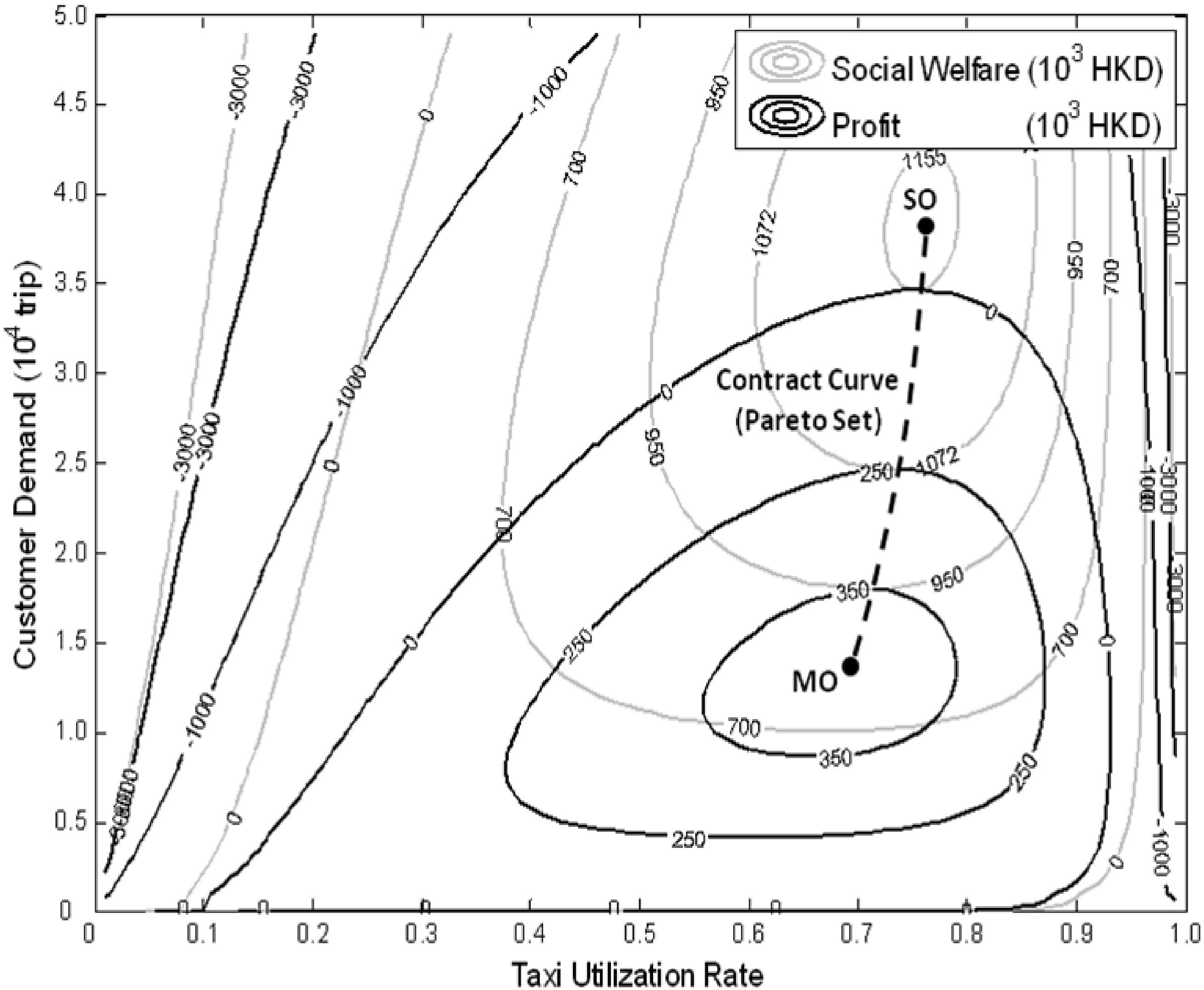
Iso-Social Welfare and Iso-Profit Contours with Constant Returns to Scale (A_0=10.0; $\alpha_1 = \alpha_2$=0.5)



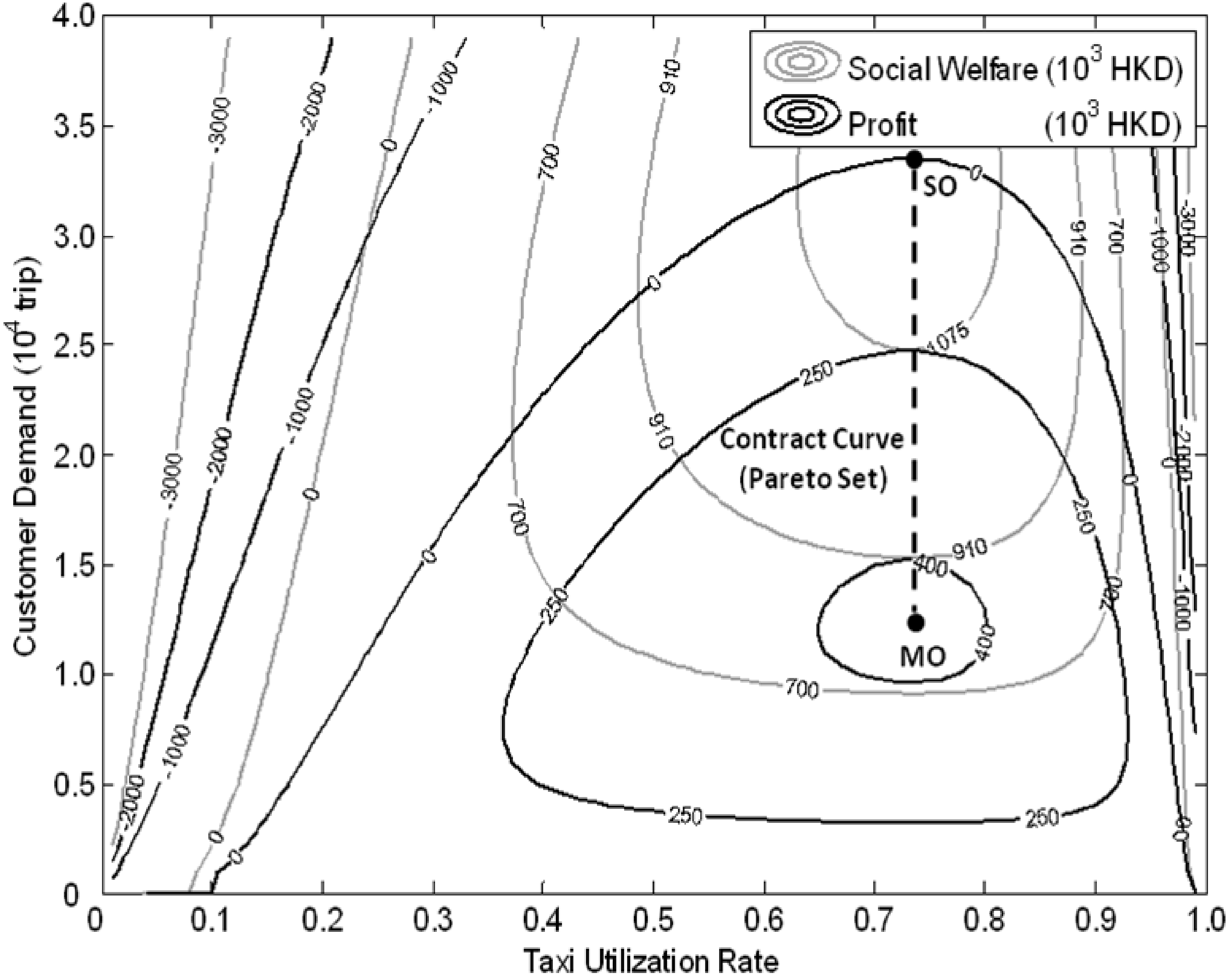
Iso-Social Welfare and Iso-Profit Contours with Decreasing Returns to Scale (A_0=200.0; α_1=α_2=0.25)



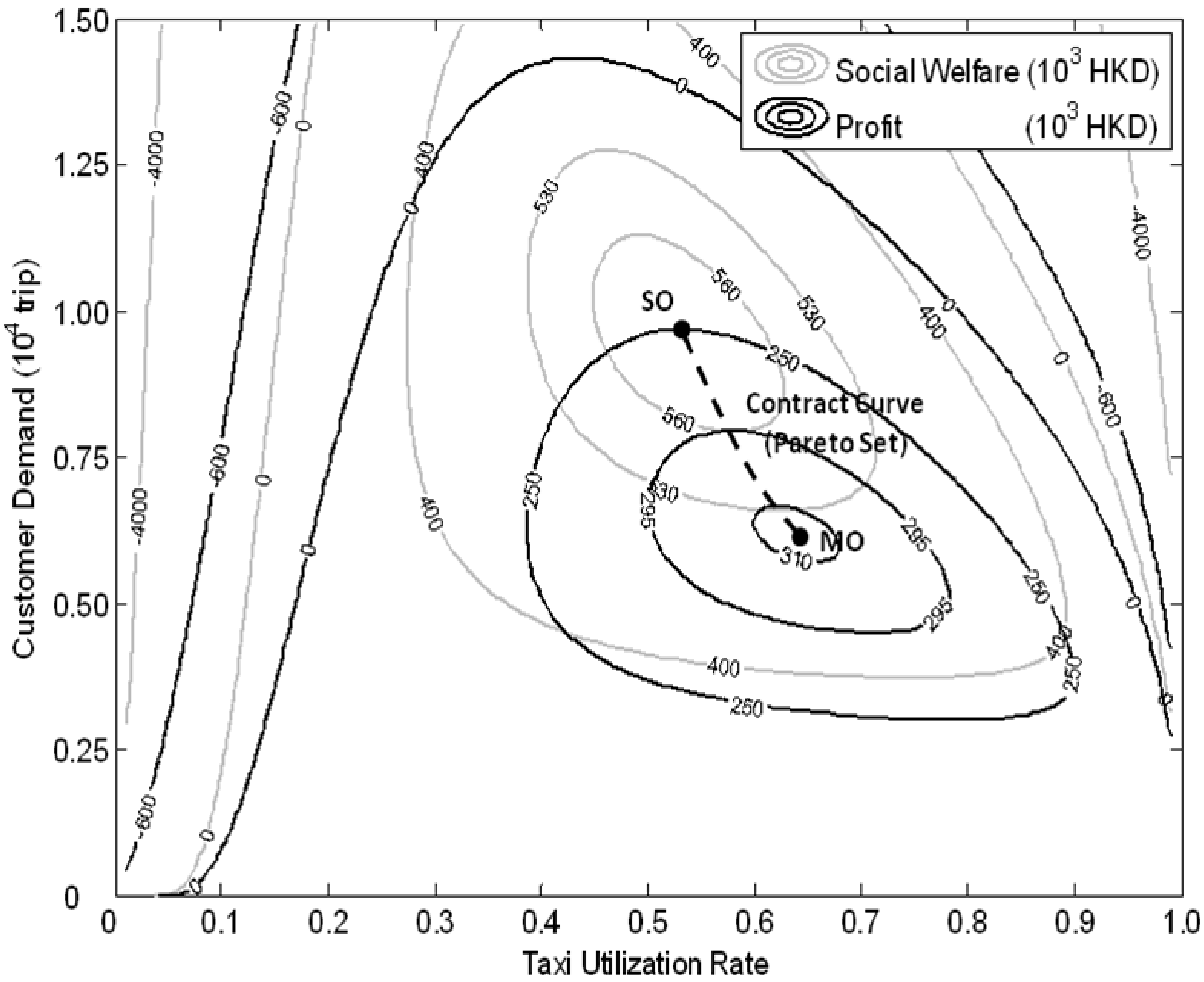
Pareto-efficient solution Set with Increasing Returns to Scale (A_0=0.2; α_1=α_2=0.75)



Pareto-efficient solution Set with Constant Returns to Scale (A_0=10.0; α_1=α_2=0.5)



Pareto-efficient solution Set with Decreasing Returns to Scale (A_0=200.0; α_1=α_2=0.25)



Conclusions

- Proved that the Pareto-improving win-win situation in terms of both taxi service quality (customer waiting time) and average taxi profit with taxi fleet size can occur only if the meeting function exhibits increasing returns to scale;
- Proved that taxi profits at social optimum are not necessarily negative, depending on the returns to scale in the bilateral taxi-customer meeting function;
- Proved that an efficient market requires that the total vacant taxi operating cost equals total customer waiting time cost for a symmetric meeting function (otherwise, multiplied by a asymmetric factor).
- Investigated the properties of the Pareto-efficient solution of social welfare versus taxi profit maximization under constant and non-constant returns to scale; and found that taxi utilization rate and taxi service quality are constant along the Pareto-efficient frontier and equal to the socially optimal levels.

Understanding these properties is helpful for efficient regulation/deregulation of the taxi market, such as price control and entry restriction!

Future Research

With the model and results established, possible future extensions include:

1. *Investigation and identification of various regulatory regimes such as entry restriction or price control for achieving Pareto-efficiency;*
2. *Bounding the efficiency (inefficiency) of market regulation (deregulation) in terms of social welfare gain (loss);*
3. *Incorporation of congestion due to both occupied and vacant taxi movement together with normal traffic into the modeling framework.*