

# OD Estimation with Inclusion of Link Flow Inequality Constraints

## リンク交通量に関する不等式制約を有する OD 推定手法

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### 1. Introduction

Several researchers have developed OD estimation methods that utilize traffic counts. In many developing countries whose main transport mode is by road, traffic officers usually control traffic during congestion. These personnel know a great deal of network information that could be of potential use in OD estimation. This study therefore seeks to investigate the potential usage of this information by generating link flow constraints, (LFC's) that could supplement the observed link flows utilized in the Holm et al classical gravity model for OD estimation.

### 2. Classical gravity model (Holm et al, 1976)

The Holm et al approach gravity model can be expressed mathematically in its general form as;

$$T_{ij} = \alpha O_i D_j f(C_{ij})^\beta \quad (1)$$

Where  $T_{ij}$  is the demand from origin  $i$  to destination  $j$ ,  $O_i$  and  $D_j$  represent zonal factors of production and attraction respectively and  $C_{ij}$  represents the travel cost function from  $i$  to  $j$ . Parameter  $\beta$  is the travel impedance parameter. The travel demand is assigned to a network to generate link flows which are then compared to the observed link flows and the impedance parameter together with the constant of proportionality,  $\alpha$  are calibrated. This procedure is repeated until a stable proportionality parameter is obtained.

### 3. Extension of Holm et al Approach

Assume the link flow constraint(s) to be satisfied in the estimation is in the form of link volume on link a,  $v_a$  is known to be less than link volume on link b,  $v_b$  i.e.  $v_a < v_b, \forall (a, b) \in Q$  where  $Q$  describes the set of link pairs with inequality constraint. However, we obtain  $v_a > v_b$ , from the first assignment. We therefore raise the cost of link  $a$  iteratively to reflect that it is less attractive to drivers than our initial estimate suggests. We repeat this until the constraint is satisfied. To

achieve this, we create an inner loop within the Holm et al approach to raise the cost of link a,  $C_a$  iteratively basing the cost increment on the link flow difference,  $\delta_v$  between  $v_a$  and  $v_b$  from the previous iteration as in the following. We presume a BPR function for link costs as in (2).

$$C_a = C_a^o + 0.15C_a^o \left(\frac{v_a}{cap}\right)^4 \quad (2)$$

Define link flow differences as  $\delta_v$  we then obtain our link cost correction  $\delta_c$  as in (4).

$$\delta_v = (v_a - v_b) \quad (3)$$

$$\delta_c = 0.6C_a^o \left(\frac{v_a^3}{(cap)^4}\right) \delta_v \quad (4)$$

In each inner loop  $k$  we then update the cost of link  $a$  before re-assignment;

$$C_a^k \leftarrow C_a^k + \delta_c^k \quad (5)$$

This procedure is illustrated as shown in the Figure 1 below with the red highlight showing the inner loop addition.

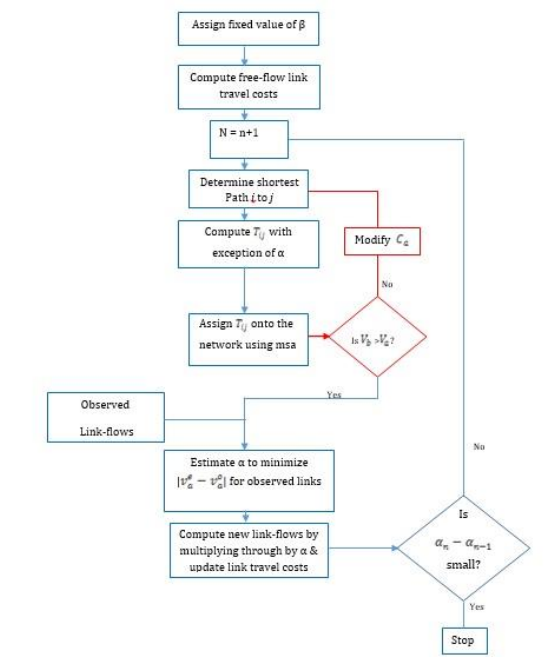


Figure 1- extension of Holm et al approach

4. Hypothetical network study

The procedure described above was applied to a three by three hypothetical network shown in Figure 2 below with e.g. “Link 01,15, 80” indicating that the link number from Node 1 to Node 2 is 01 and the reverse direction is Link 15, with both links having a free flow travel cost of 80.

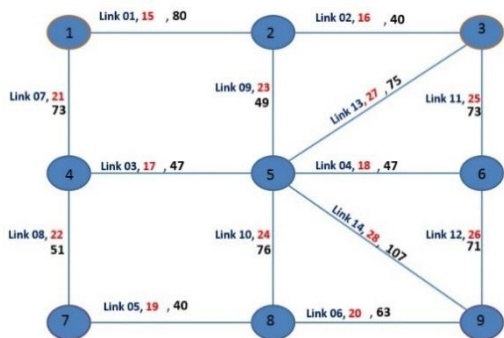


Figure 2. Hypothetical network

Trip demands from an assumed hypothetical OD matrix,  $t^o$  that fits the classical gravity model in equation 1 were assigned onto the network to generate true link flows volumes.

Investigation 1: The original OD matrix was estimated with addition of one specific link flow inequality constraint to the number of observed links used in parameter calibration

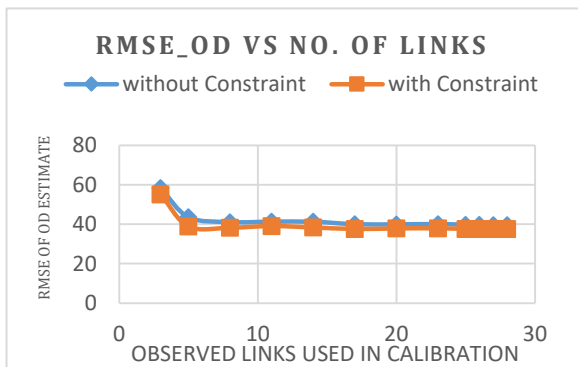


Figure 3- RMSE vs number of observed links

Result: There was a general reduction in the average OD estimate error for all the experiments run with different number of observed links as shown in Figure 3 above signifying that LFC’s can potentially reduce the average OD estimate.

Investigation 2: Perturbed matrices were used to estimate the original OD matrix using one, two and three optimally observed locations with inclusion of one and two LFC’s.

Result: A reduction was realized in the average OD estimate error with inclusion of LFC’s however, only the

first LFC did cause a significant improvement as shown in Figure 4 below.

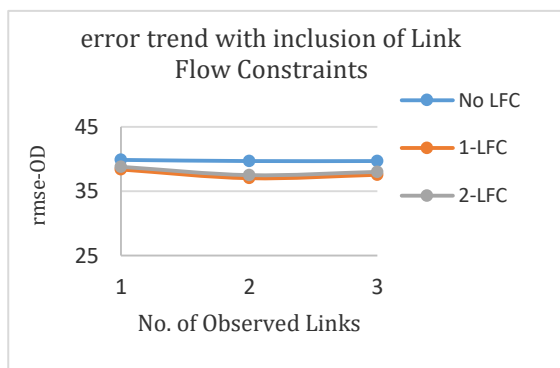


Figure 4. Error trend with inclusion of LFC(s)

5 Kampala Case Study

This study was extended to Kampala city network and the original zonal trip generations and attractions were perturbed to mimic data collection errors according to the relation;

$$O'_i = O_i + z(\gamma O_i) \tag{6}$$

The true OD matrix was then estimated using seventy five observed locations with the inclusion of one selected LFC.

Result: The OD estimate errors obtained without inclusion of the LFC was 42.3 while the error obtained with inclusion of the link flow constraint was 385.7. which was higher than the error without the LFC. This reveals that the redistribution of trips within a larger network does not necessarily result into a reduction in OD estimate error because of several possible re-distribution patterns that could meet the link flow constraint but at the same time still remain far away from the true OD matrix. The graph below shows the inner loops vs the volume difference between the LFC

5. Conclusions

A methodology is developed to include LFC’s in the classical gravity OD estimation method in this thesis. LFC’s have the potential to improve on the accuracy of the OD estimate but may be more applicable to smaller networks. The utilisation of more than one LFC appears to have no further significant impact on the OD error.

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