CASPT 2018 Extended Abstract

Using Continuous Approximation for Service Quality and Fare Level Optimisation

Zhihua JIN, Jan-Dirk SCHMÖCKER and Saeed MAADI

Abstract This paper analyses the interaction between fares and public transport service quality. The rationale is that with higher fares the operator has more resources to provide a better service. Demand in turn will depend on both service quality and fare leading therefore to the question as to whether there is an optimal fare for different general types of cities. The model developed in this paper builds on the work of Daganzo (2010). Daganzo determines optimal network headway, stop spacing as well as the ratio of a central dense PT service area compared to the whole city size. The model input is kept at its minimum considering city size, average speed of services, population, the quality of an alternative service as well as fare sensitivity. In contrast to Daganzo we include fare and demand elasticity. With this it is possible to find some general insights for a range of scenarios what type of fare levels are favorable. We focus on a flat fare structure. It is found that in such a fare structure, from the viewpoint of maximizing social welfare, a minimum, low fare would be the best. However, if the operator cost coverage ratio is considered as objective function then there exists an optimal fare above the minimum fare. We discuss further for what type of cities acceptable cost coverage ratios are considered as well as illustrate a fairly complex interaction between the decision variables.
Keywords: Public transport, Fare level, City characteristics, Network optimization, Continuous Approximation

Zhihua Jin
Kyoto University
C1-2, Katsura Nishikyo-ku, Kyoto 615-8540, Japan
E-mail: zjin@trans.kuciv.kyoto-u.ac.jp
Jan-Dirk SCHMÖCKER
Kyoto University
C1-2, Katsura Nishikyo-ku, Kyoto 615-8540, Japan
E-mail: schmoecker@trans.kuciv.kyoto-u.ac.jp
Saeed MAADI
Kyoto University
C1-2, Katsura Nishikyo-ku, Kyoto 615-8540, Japan
E-mail: maadi@trans.kuciv.kyoto-u.ac.jp

1 Introduction

There appears to be little agreement as to what fares are appropriate, fair and/or optimal. In a recent report Schmöcker et al. (2017) also describe that the definition of "fair fares" varies across cities and transport authorities.

The objective of this paper is to contribute to this discussion by showing how fares, at least under a number of simplifying assumptions, would lead to different service quality levels and with it different demand levels. We consider that the answer to this will to a large part depend on the city parameters. We aim to provide some guidance as to what fares, PT service quality and demand levels can be derived in a range of cities. More specifically as input and city parameters we vary size, population density and the demand level. Building on the work of Daganzo (2010), where the variables that we vary are optimal network headway, stop spacing as well as the ratio of a central dense PT service area compared to the whole city size. These variables together with the fare level and the demand, which we presume to be elastic, interact and provide us with indices of social welfare and subsidy needed for the operator.

2 Problem Description and Model Formulation

2.1 Problem description

Daganzo's model is considering fixed demand as his main interest was the understanding of what type of service should be operated under such conditions. Thus his model did neither include fare as it can be dropped in a model aiming to maximise unweighted user and operator utility. In contrast we consider the summation of the disutility of a fixed population, including those who might not be using public transport if it becomes too unattractive or expensive. We focus on one PT mode, bus, as it is the most prevailing one, and consider an additional alternative mode which passengers would choose instead of PT in case it is more attractive. As a potential proxy and limiting case for "rejected journeys" we consider taxi, though depending on car ownership, distance and other factors obviously private vehicles or active transport, such as walking or cycling, would be chosen by many travelers. Our choice of taxi might be seen as conservative estimate and a lower limit for demand elasticity. That is, if taxi is more attractive than the public transport service provided, then clearly the public transport option is unattractive.

2.2 Model formulation

We aim to find the flat fare that minimizes the total social disutility of a PT operator and the total travelling population. This is expressed as

$$\mathrm{z} = \lambda^{PT}(\tilde{z}_o + \tilde{z}_p) + (\lambda^{total} - \lambda^{PT})(\tilde{z}_x + \frac{f_x}{\mu}) \qquad (1)$$

In the above equation z denotes the total social disutility for the whole population λ^{total} of concern. $\tilde{z}_o$ stands for the disutility of the public transport operator per

passenger obtained from Daganzo's model, while $\tilde{z}_p$ stands for the disutility of a public transport user also obtained from Daganzo's model. $\tilde{z}_x$ is the disutility of those who chose to use our alternative mode taxi. We separate in the above formulation the public transport fare term from the total operator and passenger disutility as this is our main variable of concern. λ^{PT} denotes the actual demand for public transportation, and f_x, the fare for the proxy mode. μ is the time value, to convert all disutilities, including operator costs, into a single time equivalent unit.
The disutilities are obtained by following equations for public transportation operators and users respectively:

$$\tilde{z}_o = \pi_V V + \pi_M M + \pi_L L \tag{2}$$
$$\tilde{z}_p = A + W + T + \delta/v_w e_T \tag{3}$$

Both Equations (2) and (3) are based on Daganzo's model. Equation (2) stands for the disutility of operators per PT passenger, derived by the sum of fixed cost and operational cost. L is the summation of the infrastructure length in the periphery and in the center, to approximate fixed capital costs; and parameters the total vehicular distance traveled (V) and vehicular hours traveled in rush hour (M) reflect the operational cost. Equation (3) similarly adds up the walking access time (A), waiting time (W), travelling time (T) and penalty from transfer ($\delta/v_w e_T$), as the disutility of each PT passengers. δ stands for the weight of transfer time, while v_w indicates walking speed and e_T the expected number of transfers.

$$\tilde{z}_x = \frac{E_x}{v_x} \tag{4}$$

Equation (4) on the other hand obtains the disutility of our newly introduced taxi mode as distance traveled divided by velocity. It is assumed that the proxy disutility does not include walking access/egress time, transfer penalties nor waiting time. Ignoring the former two seems realistic for a taxi type service. Average waiting time until a taxi arrives could be added as fixed term in the utility function though for simplicity we omit it. Furthermore, since all costs are included in a single disutility function average waiting would not be distinguishable from a fixed fare that is part of $f_x$as we discuss later. To reflect that the attractiveness (and availability) of taxi will reduce with more demand we consider its speed v_x to be a function of demand.

We assume a uniform demand distribution where all OD pairs are equally likely which is clearly not realistic but instead can be considered the worst case for public transport as Daganzo also notes. Any more concentrated demand will make the case easier for public transport. Therefore our model can be considered as a "lower limiting case". Given these assumptions, optimization is done with respect to four decision variables: α denoting the proportion of the square city center with a grid PT network as in Fig,s 1 and 2, s determining the grid size and with it the distance between stops,

H for headway as well as fare f. The input parameters that are utilized in above equations are D denoting the length of the square city, v_c denoting commercial speed of vehicles, τ, the time lost per stop due to the required door operation, deceleration and acceleration; and the time added per boarding passenger, τ' (hr/p). (If the effect of alighting is significant, it can be usually subsumed into τ'.) In users' disutility calculation, v_w represents the walking access speed.

All of the remaining model parameters except for demand λ^{PT} are determined as below following Daganzo. Eq. 5 estimates the total distance operated as a function of city size as well as decision values α and s. For the determination of vehicles operated as in (6) further the third decision variable headway is used. Eq. 7 provides how the peak hour velocity is obtained while in Eq. 8 peak hour traveled time is formulated. Eq. 9 to 12 formulation are from Daganzo's paper for passengers' disutility.
For the model we further need to obtain the proportion of passengers that travel within the city center, within the periphery and those travelling within both parts of the city in order to obtain the expected distance travelled in the network. Since Daganzo's split of the travelling groups does not appear fitting to us for some fare scenarios that we would like to extend further, we derive all of the expected values for vehicular travelling distance (E) in the appendix provided in the full paper.

$$L = \frac{D^2(1+\alpha^2)}{s} \tag{5}$$

$$V = \frac{2D^2(3\alpha - \alpha^2)}{sH} \tag{6}$$

$$\frac{1}{v_c} = \frac{1}{v} + \frac{\tau}{s} + \frac{2.5(1+e_T)\tau' \lambda_{PT} sH}{(3\alpha - \alpha^2)D^2} \tag{7}$$

$$M = \frac{V}{v_c} \tag{8}$$

$$e_T = 1 + \frac{1}{2}(1-\alpha^2)^2 \tag{9}$$

$$A = \frac{s}{v_w} \tag{10}$$

$$W = \left[\frac{2+\alpha^3}{3\alpha} + \frac{(1-\alpha^2)^2}{4}\right]\mathrm{H} \tag{11}$$

$$T = \frac{E}{v_c} \tag{12}$$

$$E_{PT} = \frac{\mathrm{D}(12 - 9\alpha - 9\alpha^2 + 23\alpha^3 - 5\alpha^4 - 5\alpha^5 + 2\alpha^6 + \alpha^7)}{12} \tag{13}$$

The public transport demand λ is obtained depending on the relative disutility of public transport and taxi considering a base demand for public transport referred to as λ^{PT0} We test both linear as well as logit formulations for demand.

For the linear model, Figure 1 illustrates the demand development depending on the disutility and fare for taxi. Given the original fare and disutility, we assume that all

demand is utilizing the PT. We set this demand as upper limit, so that we also refer to it as “potential public transport users”.
As there are people who would not take PT anyway, regardless of fare, we set an upper bound for PT demand as flat part of the curve marked with the upper marked point reflects, which is the sum of all PT and Taxi users. When the sum of network average fare and travel disutility equals to that of PT, half of the passengers would turn to taxi, as the green line marks.

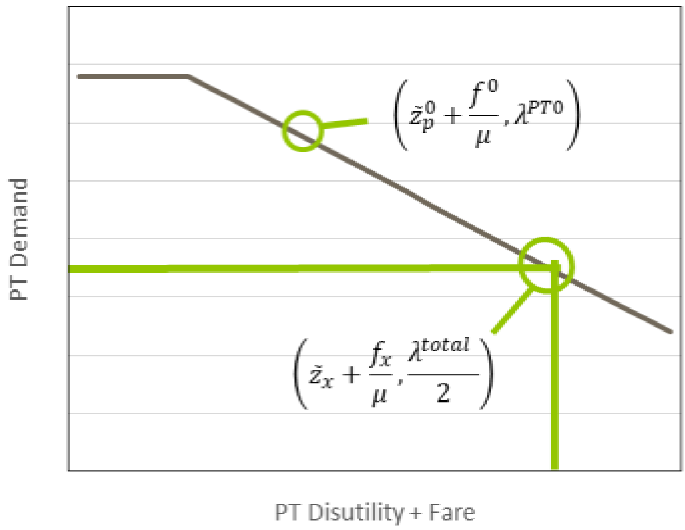

Figure 1. Illustration of PT demand elasticity with the linear model

Assuming these two reference points, i.e. PT demand λ^{PT0} at current situation and PT demand being halved in case of PT disutility equals taxi disutility in the linear model can be derived:

$$\lambda_{\mathrm{PT}} = \begin{cases} \lambda^{PT0}\left(1 - \dfrac{\tilde{z}_p + \frac{f}{\mu} - \tilde{z}_p^0 - \frac{f^0}{\mu}}{2\left(\tilde{z}_x + \frac{f_x}{\mu} - \tilde{z}_p^0 - \frac{f^0}{\mu}\right)}\right) & \text{if } 2 - \dfrac{2\lambda^{\mathrm{total}}}{\lambda^{PT0}} \geq \dfrac{\tilde{z}_p + \frac{f}{\mu} - \tilde{z}_p^0 - \frac{f^0}{\mu}}{\tilde{z}_x + \frac{f_x}{\mu} - \tilde{z}_p^0 - \frac{f^0}{\mu}} \\ \lambda^{total} & \text{if } 2 - \dfrac{2\lambda^{\mathrm{total}}}{\lambda^{PT0}} \geq \dfrac{\tilde{z}_p + \frac{f}{\mu} - \tilde{z}_p^0 - \frac{f^0}{\mu}}{\tilde{z}_x + \frac{f_x}{\mu} - \tilde{z}_p^0 - \frac{f^0}{\mu}} \end{cases} \quad (14)$$

In addition to service quality and travel disutility as another indicator to evaluate specific fare scenarios we consider the percentage of operational costs (including fixed infrastructure costs) covered by the fare revenue as in (18). Most operators face such constraints in order to be able to on the one hand provide good public transport services but on the other hand not overextend their service if the service is too low. In the scenarios described in the full paper ϕ will help us to generally show for which type of cities larger subsidy requirements can be expected.

$$\phi = \frac{f_{PT}}{\tilde{z}_o} \tag{15}$$

3 Experiments and results

First of all, to understand the overall tendency of current situation, as well as to see how it would be better according to this model, we compare the values of input variables as well as each parameters we assumed. The result is shown as follows. Parameters in the initial scenarios are in line with Daganzo's Barcelona related example.

Table 1. Absolute value comparison for current situation and the realistic optimum situation at base scenario

	Daganzo (2010), with fares t	Optimum
α	0.65	0.65
s [km]	0.45	0.44
H [min]	4.17	3.90
$\tilde{z}_o$ [min]	6.95	6.63
$\tilde{z}_p$ [min]	42.54	42.14
$\tilde{z}_x$ [min]	16.00	10.66
z [min]	62468	18577
fare[$]	2.29	1.47
PT demand [pax/min]	333.33	380.95
Φ	0.99	0.67

As in the table, the fare at the optimum would be lower than the current situation, while the headway is even lower as well. This increases the number of passengers, in fact all prospective passengers, to PT with lower passengers' disutility. This is also the reason why the disutility for operators per passengers could remain slightly lower than current despite the drop in headway.

Obviously if the coverage from revenue, ϕ is considered as a rather important aspect, the optimum situation is not as favorable as the current ones for the operators. Yet from a social welfare perspective, the optimum situation provides the lowest total disutility. In the full paper we will discuss further scenarios showing and explaining the complex triangular interaction between fare, service level and PT demand by varying the minimum fare that can be charged as well as other input parameters such as total demand, city size and taxi fare.

Key References

Daganzo, C.F. (2010). Structure of Competitive Transit Networks. Transportation Research Part B, 44(4),, pp. 434–446, 2010.

Schmöcker, J.-D, Fonzone, A., Maadi, S. and Van der Ploog, R. (2017). Determining Fare Structures: Evidence and Recommendations from a Qualitative Survey. EMTA Brief. Available from < http://www.emta.com/spip.php?article693>.