Advanced discrete choice models with applications to transport demand

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Topics

- Model specification
- Model estimation
- Interpretation of results
- Use of advanced models in practice

Topics

- Model specification
- Model estimation $\Rightarrow$ strongly related
- Interpretation of results
- Use of advanced models in practice

Model specification I: correlation structure

- Standard models assume covariance homogeneity
- Mixed covariance structures

$$P_n(i) = \int_{\boldsymbol{\lambda}} P_n(i \mid \boldsymbol{\lambda}) \, f(\boldsymbol{\lambda} \mid \Omega) \mathrm{d}\boldsymbol{\lambda}$$

- Allowing for covariance heterogeneity
  - Improvements in model fit
  - Differences in forecasts
  - Differences in trade-offs

Mixed Covariance example

	MNL	NL	MCOV_U	MCOV_N
Null LL	-3517.56	-3517.56	-3517.56	-3517.56
Final LL	-2329.07	-2320.23	-2191.42	-2189.8
Par	14	15	16	16
Adj.ρ^2	0.3339	0.3361	0.3725	0.3729
VTTS_{car}	54.64	46.03	42.43	41.96
VTTS_{PT}	23.64	27.25	30.13	30.94

Changes in market shares after 20% inc. in base TT

	MNL	NL	MCOV_U	MCOV_N
base alt.	-13.33%	-18.54%	-17.96%	-18.51%
early dep.	30.72%	46.17%	51.01%	53.36%
late dep.	31.27%	47.51%	39.22%	40.39%
PT	26.27%	17.81%	15.62%	14.93%

Model specification II: taste heterogeneity

- Continuous *vs* discrete mixtures

$$P(i) = \int_{\beta} \left[P\left(i \mid \beta\right) f\left(\beta, \Omega\right) \right] \mathrm{d}\beta$$

  - specification issue: choice of distributions

$$P(i) = \sum_{j_1=1}^{m_1} \cdots \sum_{j_K=1}^{m_K} P\left(i \mid \beta = \langle \widehat{\beta}_1^{j_1}, \ldots, \widehat{\beta}_K^{j_K} \rangle \right) \pi_1^{j_1} \ldots \pi_K^{j_K}$$

  - specification issue: selection of number of support points

Taste heterogeneity example I

Model	MNL	MMNL(N)	DM(2)	DM(3)
Respondents:	185	185	185	185
Observations:	1,421	1,421	1,421	1,421
Final LL:	-880.96	-849.65	-845.40	-844.60
adj. ρ^2:	0.1036	0.1343	0.1366	0.1354
Estimation time (s):	1	75	1	3
Mean VTTS (DKK/hour):	19.77	30.41	32.81	34.29
VTTS standard deviation	-	33.70	36.55	41.86

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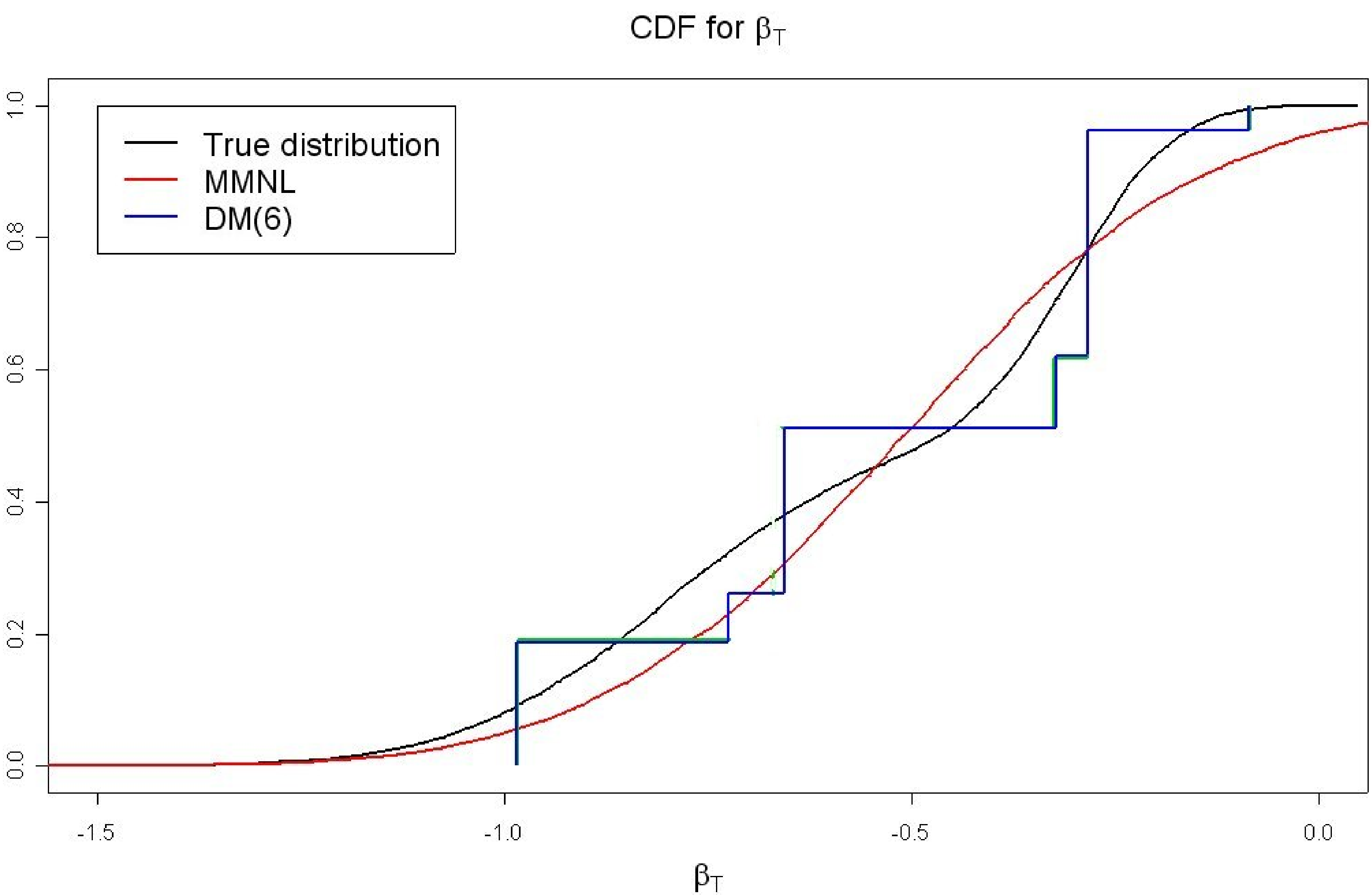
Taste heterogeneity example II



Model specification III: joint structures

- Correlation & random taste heterogeneity
  - phenomena taking place in unobserved part of utility
- Both processes can act at the same time
  - Solution:
    use GEV mixture models, or joint RCL-ECL formulation
- Majority of applications account only for one of two phenomena
  - Big risk of confounding
    - wrongly retrieved taste heterogeneity: issues in CBA, etc
    - wrongly retrieved correlation: issues in forecasting

Confounding example

	True model	MNL	NL (rail-SM)	NL (rail-car)	RCL	NL mixture
Final LL		-1562.29	-1560.55	-1525.55	-1163.73	-1147.78
adj. $\rho^2(0)$		0.5239	0.5241	0.5347	0.6433	0.6478

	True model	MNL est.	MNL t-stat.	NL (rail-SM) est.	NL (rail-SM) t-stat.	NL (rail-car) est.	NL (rail-car) t-stat.	RCL est.	RCL t-stat.	NL mixture est.	NL mixture t-stat.
β_{TC} (μ)	-0.100	-0.035	-20.14	-0.035	-19.99	-0.028	-16.23	-0.152	-8.75	-0.093	-14.72
β_{TC} (σ)	0.035	-	-	-	-	-	-	0.061	8.64	0.037	14.38
β_{HW} (μ)	-0.020	-0.023	-14.16	-0.022	-10.98	-0.014	-9.91	-0.034	-10.05	-0.018	-9.74
β_{HW} (σ)	-	-	-	-	-	-	-	0.009	1.97	0.001	0.49
$\beta_{TT,car}$ (μ)	-0.030	-0.021	-14.15	-0.020	-13.51	-0.015	-11.83	-0.054	-7.78	-0.028	-10.05
$\beta_{TT,car}$ (σ)	-	-	-	-	-	-	-	0.016	4.10	0.002	0.73
$\beta_{TT,rail}$ (μ)	-0.040	-0.032	-20.79	-0.032	-15.72	-0.022	-13.63	-0.065	-12.13	-0.037	-12.85
$\beta_{TT,rail}$ (σ)	-	-	-	-	-	-	-	0.002	2.14	0.001	1.05
$\beta_{TT,SM}$ (μ)	-0.035	-0.022	-10.13	-0.022	-10.50	-0.013	-6.28	-0.059	-9.83	-0.034	-11.61
$\beta_{TT,SM}$ (σ)	-	-	-	-	-	-	-	0.003	1.17	0.001	0.56
$\lambda_{rail,SM}$	0.50	1.00	-	0.89	1.22	1.00	-	1.00	-	0.48	4.59
$\lambda_{rail,car}$	1.00	1.00	-	1.00	-	0.49	5.92	1.00	-	1.00	-

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Model estimation I: type of draws

- Simulation-based estimation of continuous mixture models
- Various alternatives to standard PMC draws
- Halton draws:
  - Problems even with only $4 - 5$ dimensions
  - Also problems with various adapted versions
- MLHS
  - outperforms Halton sequences, but high variance
- Advanced approaches
  - difficult to implement, lack of available code
  - sometimes no guarantee of better performance

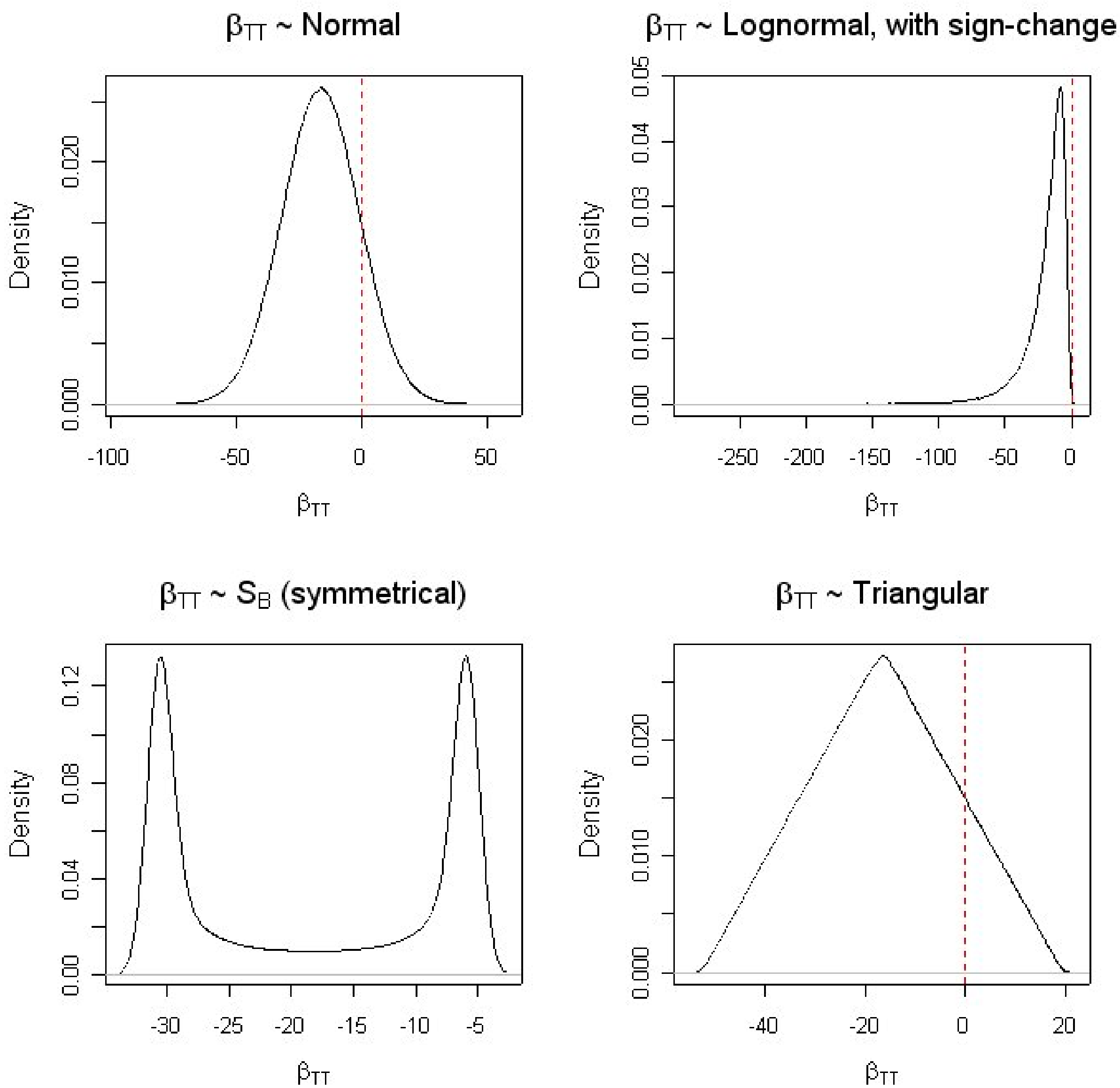
Model estimation II: number of draws

- Number of draws has very significant impacts on results
- Guidelines in literature
  - Worrying number of applications using 125 Halton draws, in up to 8 dimensions!
- Other approaches for reducing estimation cost
  - MNL starting values
  - Pre-estimation with lower number of draws
- Advisable to use multiple runs with different draws and starting values

Model interpretation

- Model specification has effects on model results
  - important to recognise in model interpretation
- Example: choice of distributions in MMNL
  - shape assumptions impact model results
- Application: Danish VOT data
  - compare models with different distributional assumptions

Model interpretation example



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Model interpretation example (continued)

- Significant differences in implied distributions

	Normal	Lognormal	S_B	Triangular
adj. ρ^2	0.1484	0.1518	0.1512	0.1486
% positive	14.53%	0%	0%	14.6%

- Differences in model fit very small

- Guidance: use flexible distributions

Theoretical rules *vs* practical considerations

- Applied part...
- Modelling air travel choice behaviour
- Joint choice of airport, airline and access mode
  - departure from state-of-the-art
  - major gains in model performance
- Three datasets
  - RP data (SF-bay area & Greater London)
  - SP data (US, internet-based)

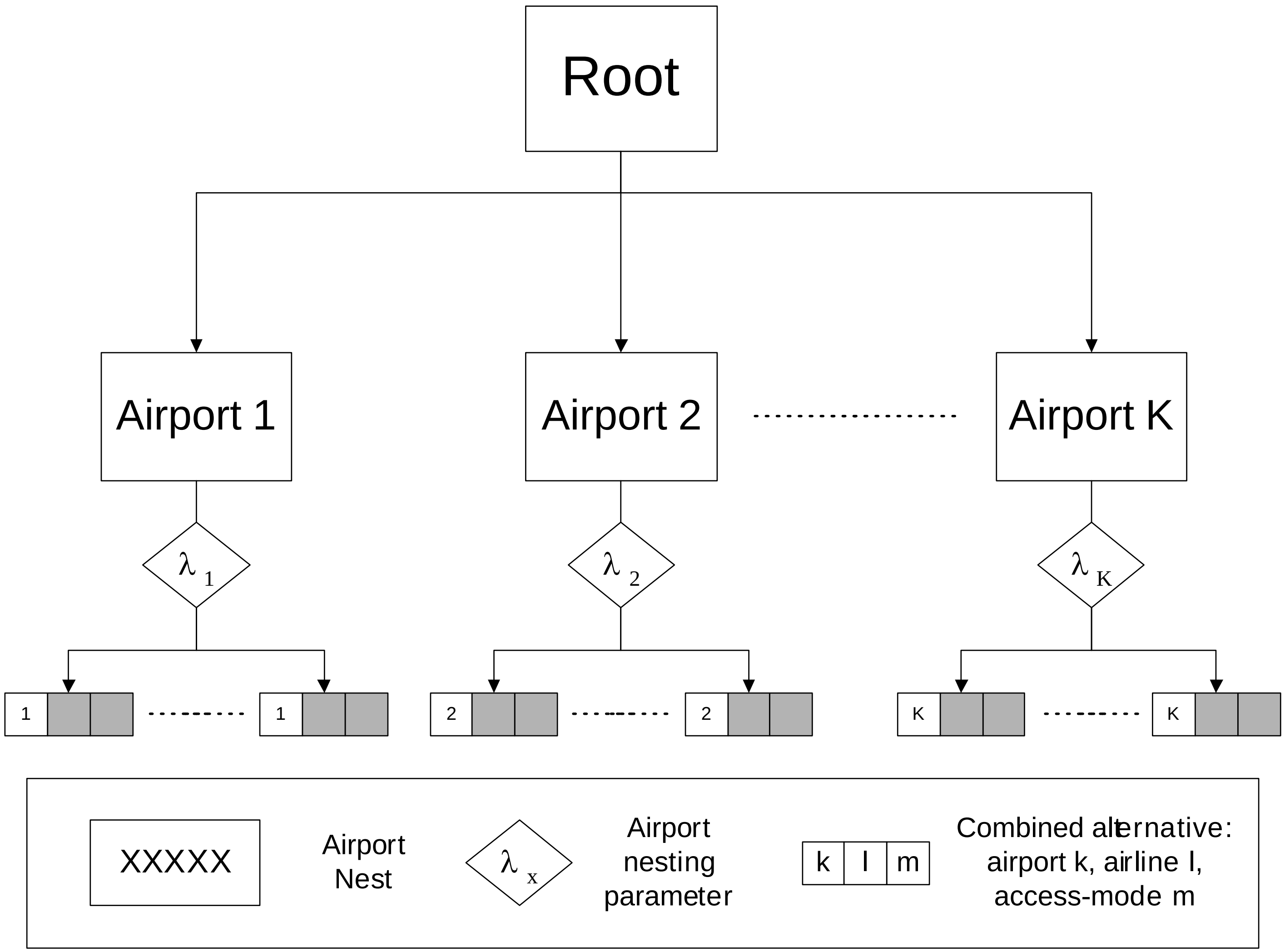
Random taste heterogeneity

- Some gains in model fit, and insights into behaviour
- Huge increases in estimation cost
- Use of Normal almost unavoidable
- Use of high number of draws almost impossible
- London data ($> 10,000$ observations, 324 alternatives)
  - basic MMNL models take in excess of 1 month to estimate
- Flexible interactions and non-linearities massively reduce scope for retrieving random taste heterogeneity

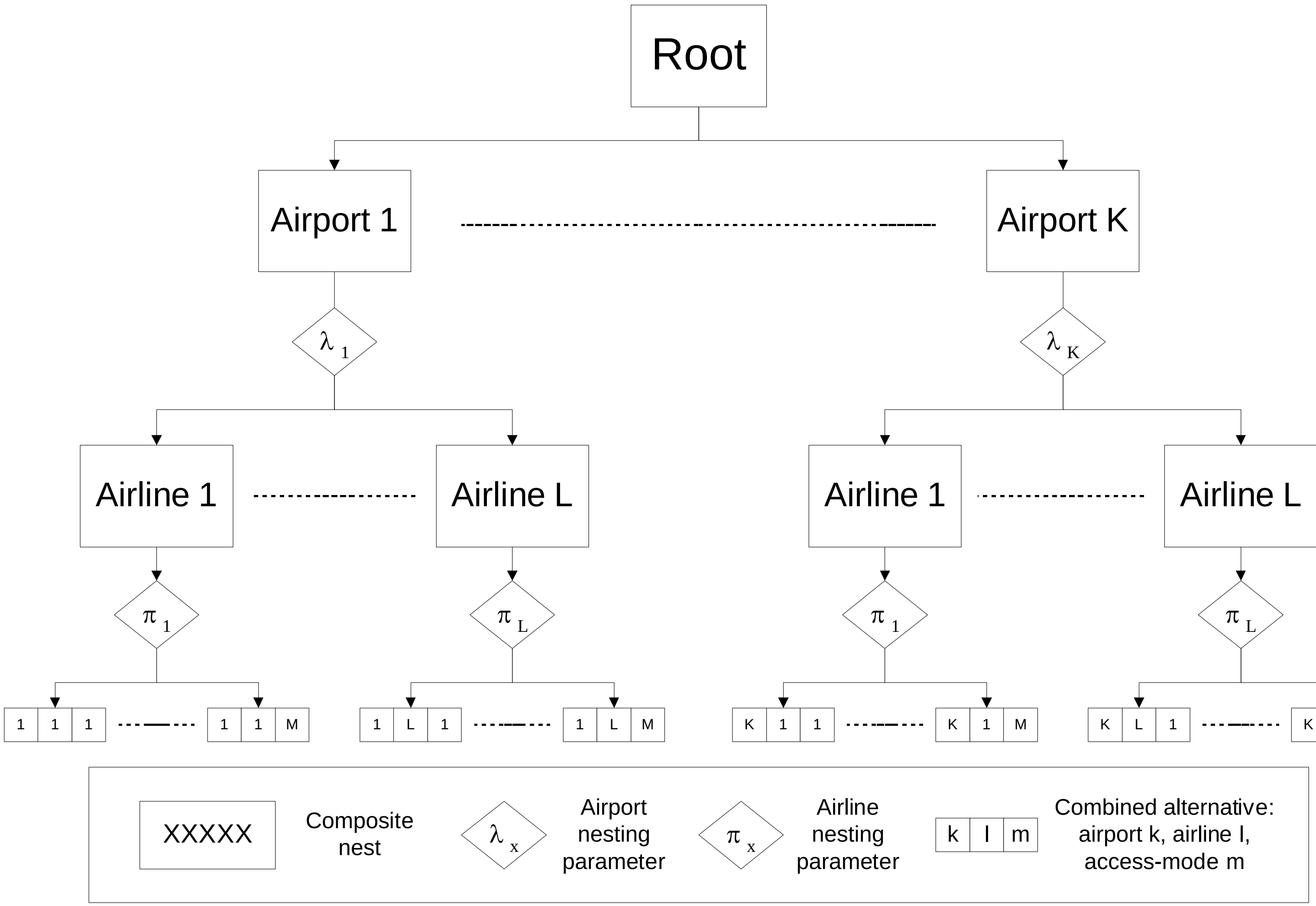
Correlation structure

- Shortcomings of NL model for multi-dimensional choice processes

2-level NL


Root
Airport 1
Airport 2
Airport K
λ 1
λ 2
λ K
1
1
2
2
K
K
XXXXX
Airport Nest
λ x
Airport nesting parameter
k
l
m
Combined alternative: airport k, airline l, access-mode m

3-level NL



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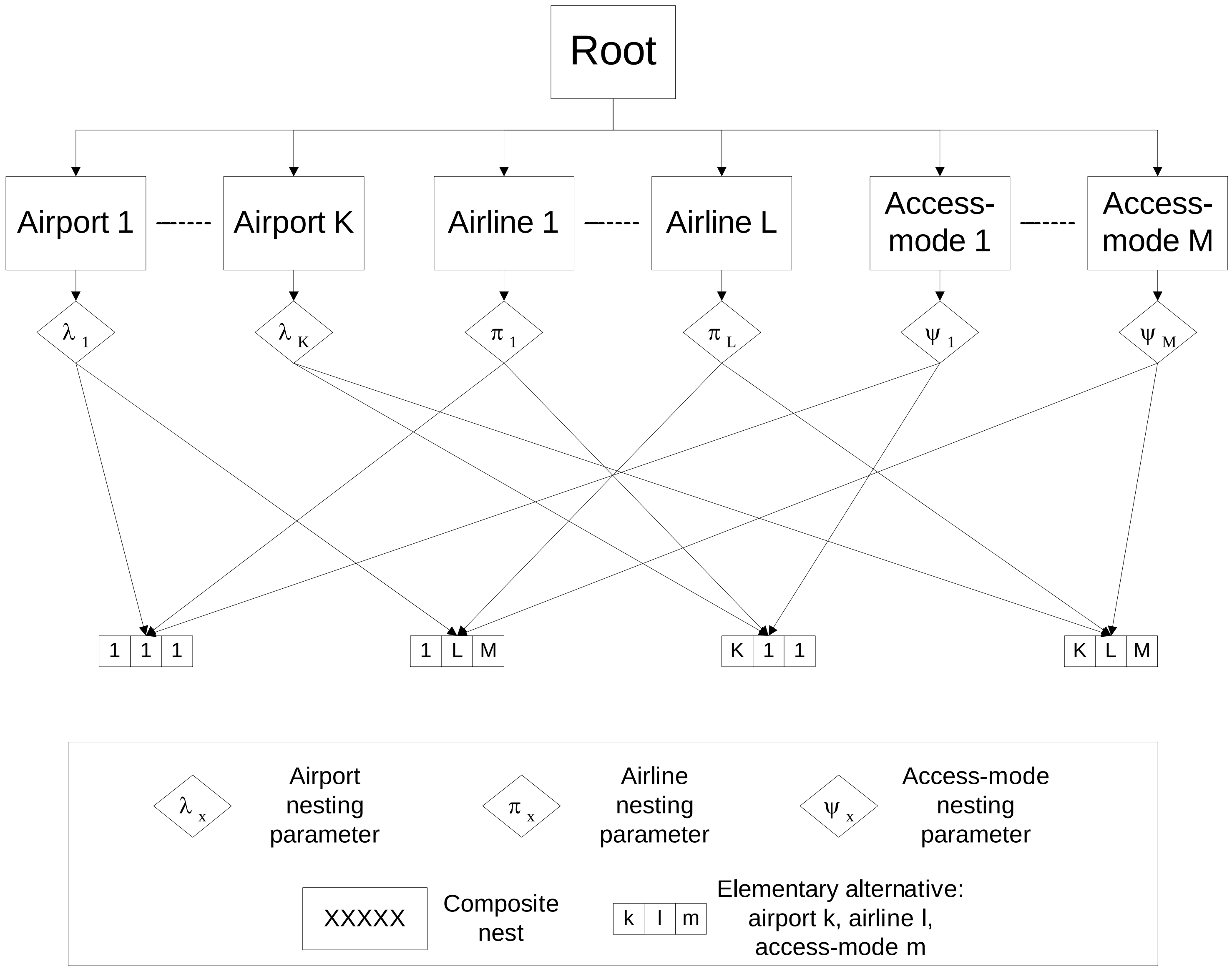
CNL


Root
Airport 1
Airport K
Airline 1
Airline L
Access-mode 1
Access-mode M
λ 1
λ K
π 1
π L
ψ 1
ψ M
1 1 1
1 L M
K 1 1
K L M
λ x
Airport nesting parameter
π x
Airline nesting parameter
ψ x
Access-mode nesting parameter
XXXXX
Composite nest
k l m
Elementary alternative: airport k, airline l, access-mode m

Correlation structure

- Shortcomings of NL model for multi-dimensional choice processes
- Need cross-nesting structure

	Par	*Adj.* ρ^2	est.time
MNL	55	0.3445	minutes
NL airport	59	0.3465	hours
NL airline	74	0.3469	hours
NL access	60	0.3499	hours
CNL	91	0.3578	weeks

Advanced structures

- GEV mixtures
- Covariance heterogeneity
- Essentially impossible to use
  - Potential confounding and other mis-specification effects

Conclusions

- Major issues in specification, estimation and interpretation
- Certain guidelines
- Often need to be violated in practice

Discussion

- State-of-the-art has moved forward at great speed
- State-of-practice is trailing behind
- Advanced models often almost inapplicable
  - Estimation and application cost
  - Data requirements
- Often rather small gains in performance
  - log-likelihood is not everything
- Mixture models over-hyped
- Need to educate, and sell our models better

Thank you ...

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